Use of Calculators in the Mathematics Classroom

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Abtract: *A key variable in the use of calculators in the learning of mathematics is the teacher. In turn there are many factors that influence whether an individual teacher uses the calculator, and if they do then how they use it. This study reports on a ten-year longitudinal survey data into the use of calculators in the upper secondary school. It presents the pattern of calculator use, some possible reasons for this pattern, and obstacles to increased use. In addition the relationship between calculators and national assessment and equity are examined. Results show that many teachers see benefits in using calculators in mathematics teaching although a sizeable minority are opposed to their use. Further, there is a continuing need for professional development that specifically addresses how to integrate calculators into mathematics teaching in a manner that focuses on the mathematics.*

1. Background

In recent years research on calculator use in the learning of school mathematics has tended to move from an emphasis on student learning to the influence of the teacher (see e.g., [1]), recognising their key role in the use of calculators. In turn, there are many factors that impinge on a teacher's use of calculators, as with any technology. Among these, primary influences are teacher affective variables (such as beliefs and attitudes), their thinking about, and perceptions of, the nature of mathematical knowledge and how it should be learned, and their mathematical and pedagogical content knowledge. Other influences that need to be taken into account are social relations, institutional standards, tools, and tasks used (see [2] and [3]), attitude to, and beliefs about the technology, as well as teacher confidence and ability to use it to teach mathematics.

There has been less emphasis on research into the decision process that teachers engage in when deciding whether to use calculators, and if so, how and when to use them in learning. These decisions may be dependent on a number of factors. One of these, described in [4] is the concept of teachers' *pedagogical technology knowledge* (PTK). This has been presented as a useful way to think about what teachers need to know in order to teach well with technology. PTK includes not simply being a proficient user of the technology, but more importantly, understanding the principles and techniques required to teach *mathematics* through the technology. This necessitates a change of mindset on the part of teachers, a shift of focus to a broader perspective of the implications of the technology for the learning of the mathematics [5]. Developing PTK requires attending to the teacher's perspective on mathematics and technology, the relationship between the two, their use of technology and their personal instrumentation of it (see [6]). It also involves the teacher in the transformation of the technological tool into an instrument and differentiation of qualitatively diverse ways of employing technological tools in teaching mathematics (such as direct procedural calculation, computational check), or building conceptual knowledge of mathematics (see [7]). Teachers also need to consider the kind of tasks that they set students when using technology in order to assist students to develop simultaneously the calculator and by-hand techniques and theory (see [3]) needed to make progress in mathematics. Hong and Thomas's (see [8]) model of the influences on PTK suggests that teacher confidence in using technology is a key driver of its growth.

Orchestrating and directing all of the influences mentioned above in such a way that good classroom learning eventuates can be very challenging. Following investigation of the didactic contract of a teacher using a graphic calculator (GC) during function and limit concept lessons, delos Santos and Thomas (see [9]) concluded that the teacher has to be open to new approaches, be willing to work around constraints, be open to personal learning, and be able to reflect "on the tension created between valuing a formal, primarily algebraic approach to mathematics and an investigative style of teaching" with the GC (see [9], p. 357). Orchestrating the integration of technology in teaching involves many aspects. One described in [10] (p. 329) is the need for resequencing as teachers have "to integrate graphic calculators… and to organise, when it is possible, backward and forward motions between calculators, theoretical results and calculus by hand." Another part of this challenge involves the influence of the teacher's practice on their students. For example, Kendal and Stacey (see [11]) have shown that, in an introductory calculus course using calculators, the teachers' *privileging* of certain approaches differentially affected their students' learning. Using the metaphors of technology as *Master*, *Servant*, *Partner*, and *Extension of Self* to describe a hierarchy of roles for the interaction between the teacher and technology, the task facing teachers has been expressed by Goos, Galbraith, Renshaw, and Geiger (see [12], p. 318) in these words: "Perhaps the most significant challenge for teachers lies in orchestrating collaborative inquiry so as to share control of the technology with students."

It was against this background of integration of calculators in teaching that the current research project took place. It sought to ascertain current practice in the use of calculators in the upper secondary mathematics classroom, along with teacher and other factors that might be influencing it, especially the role of external assessment.

2. Method

Longitudinal studies, where at least two sets of data are collected from the same population over an extended time span, are relatively rare in mathematics education research. This ten-year longitudinal study, with a population of all secondary mathematics teachers in New Zealand, began in 1995, when a questionnaire on calculator and computer use was mailed to every secondary school in New Zealand. Replies were received from 90 of the 336 schools (26.8%), a reasonable response rate for a postal survey, and from 339 teachers in the schools. Some of the results of this survey were published at the time (see [13]) and were used to form a baseline comparison. This original survey was followed by a second in 2005 in order to gain longitudinal data on how the situation might have changed in schools over this period. Since 1995 teaching has become an even more stressful and demanding profession in many ways, particularly in terms of demands on time, and so teachers are more reluctant than ever to spend their valuable time filling in forms or research questionnaires. However, we had learned lessons from 1995 and stamped, addressed envelopes were enclosed for all the schools. Also the posted questionnaire was followed up several weeks later with a faxed copy. Using this approach we achieved a very good 57.4% response, from 193 of the 336 secondary schools in the country. We also received completed questionnaires from a total of 465 teachers in these 336 schools, as well as the school information. While the questionnaires sent out in the two years were not identical, due to changes in emphasis, they did have a number of questions in common. On both occasions they used both closed and open questions to provide valuable data on calculator issues such as: the number of calculators in each school; the level of access to the calculators; the pattern of use in mathematics teaching; and teachers' perceived obstacles to calculator use (Figure 2.1 has a selection of questions from the second survey). This data enables us to come to some conclusions about the changing nature of calculator use in the learning of mathematics in New Zealand secondary schools. In addition to the survey a group of 32 volunteer teachers, who volunteered via their survey response, were interviewed about their views on technology, and some of their lessons using technology were observed.

Q8	Approximately how many of each of these calculator types does your mathematics department own?	Casio	______
		Texas Instruments	______
		Sharp	______
		Other ____________	______
Q9	Approximately what % of your school's senior mathematics students own their own calculators?	Year 12	____%
		Year 13	____%
Q15	Would you like to use graphic calculators more often in your mathematics lessons?	Yes	1
		No	2
Q16	If you answered yes to question 15, what do you see as obstacles to your use of them? Please rank in order any of these which apply (ie 1 for biggest obstacle, 2 for the next, etc.).	Lack of confidence	___
		Lack of PD	___
		Calculator availability	___
		School policy	___
	Other ____________________________________	Government policy	___
		Other______________	___

Q17 With which years do you sometimes use graphic calculators in your mathematics lessons?

Year 12 ☐ Year 13 Calculus ☐ Year 13 Statistics & Modelling ☐ None ☐

Q18 With which years do you regularly use graphic calculators as an integral part of mathematics lessons.

Year 12 ☐ Year 13 Calculus ☐ Year 13 Statistics & Modelling ☐ None ☐

Q19 Do your students use calculators in their mathematics lessons only when directed by you?

Yes ☐ No ☐ Sometimes ☐ Depends on ______________________ ☐

Q20 What kinds of calculators do your students use in their mathematics lessons ?

Scientific ☐ Graphic ☐ Computer Algebra System (CAS) ☐ None of these ☐

Q22 Please give the main advantage or benefit you have found, or feel to be true, of using technology in mathematics lessons.

Figure 2.1 Sample questions from the 2005 survey (Some formatting changed)

3. Results and Discussion

The longitudinal study used the baseline data on calculator use in schools gathered by Thomas in 1995 (see [13]) in order to make comparisons with the current position and look for trends. In 1995 there was an average of 22.6 calculators (52% Casio) owned by mathematics departments and 96% of Year 12 (age 17 years) and 97% of Year 13 (age 18 years) mathematics students owned their own calculators. In 2005 the average number of calculators owned by a mathematics department was 45.7 (of which 68.6% were Casio, 14.4% Texas Instruments, and 15.4% Sharp). In Year 12, 86.4% and Year 13, 87.9% owned their own calculator, which interestingly represented a drop on the 1995 figures. It was noteworthy that in 2005 the calculator types owned were: scientific 76.1%, graphic calculator (GC) 27.1% and CAS 0.2%. The low CAS use is no doubt a reflection of the fact that they were not allowed in external assessment at the time of the survey.

From the survey for all secondary school teachers and heads of departments (HOD's), 75.5% of respondents who teach year 12 classes said that they sometimes used GC's in their lessons. However when they were asked as to whether they regularly used them, this number dropped to just under half (49.4%). Among teachers of year 13 calculus, 91.8% sometimes use GC's, while 75.4% regularly use them. Among teachers of year 13 statistics, 79.4% sometimes use GC, while 66.7% regularly use them. During 1995 mathematics lessons 75.8% of Year 12 and 62.5% of Year 13 regularly used calculators as an integral part of the lessons with 69.8% using them at least once a week, 14.2% at least once a month, and 9.2% at least once a term. These figures represent a drop in regular use of calculators in years 12 and 13, especially in year 13 calculus lessons. This is surprising since this course contains a lot of graphical work on functions that would appear to lend itself to GC work. In 1995 6.2% of Year 12 and 5.0% of Year 13 used the calculator only when directed by the teacher compared with a total of 10.2% in 2005. Thus the majority of teachers surveyed said that students were not using calculators in their lessons only when directed by them, indicating most students use calculators when they decided to, without the direction of a teacher. The question of whether it is better for students to own their own technology or for the school to provide it was specifically addressed in the questionnaire. 66.0% of the teachers agreed that student ownership was the best situation, with only 14.8% disagreeing. The two clear benefits from students having their own technology are improved access and lowering of the pressure on already over-committed department and school budgets. The questionnaire revealed that only 10.3% of mathematics departments have a technology budget, and the average size of these is NZ$2762.50 per year.

There has been quite a lot said, often in the media by parents and others, about the possible negative effects of calculator use in the mathematics classroom and we wanted to know what the opinion of the teachers was on this subject. The responses to the question of whether calculators 'may be' (1995) or 'are often' (2005) detrimental to students' mathematical understanding are given in Table 3.1.

Table 3.1 Teachers' Views on Whether Calculators May be Detrimental to Understanding

	1995 RESPONSE % (*N*=339)	2005 % RESPONSE % (*N*=464)
Strongly Agree	4.7	5.0
Agree	20.1	21.6
Neutral	14.2	18.8
Disagree	35.1	33.1
Strongly Disagree	25.1	14.0
No Response	0.9	7.5

The summary shows that in 1995 24.8% of teachers agreed that calculators may be detrimental, and in 2005 26.6% thought that they often are. In the same period the number disagreeing dropped from 60.2% to 47.1% (χ^2=13.7, $p<0.001$). It seems that what has happened in the intervening years has done nothing to alleviate the perception of a significant minority of teachers that calculators may be more damaging than useful to student understanding. In fact, on the basis of this question, there is some evidence that the situation has changed so that fewer teachers are convinced that calculators are never detrimental.

However, when the teachers were asked whether they agreed with the statement that ‘calculators’ (1995) or ‘technology’ (2005) are/is of little benefit in mathematics teaching, we see from Table 3.2 that there was a large majority disagreeing; 87.3% in 1995 and 75.6% in 2005, although this too has fallen. Since the calculator is the technology most often used in classrooms, one possible explanation for why many teachers see the calculator as of value but why some also think that it can be detrimental is that it depends on the way in which it is used in teaching. This would agree with the argument that whether the calculator is beneficial or harmful to learning depends on how it is used. Hong, Thomas and Kiernan (see [14]) found that weaker students can become dependant on the calculator to the point where there mathematics is weaker when they don’t have access to one, and similarly [15] reports that some students who were not motivated by technology nevertheless became dependent on it.

Table 3.2 Teachers’ Views on Whether Calculators are of Little Benefit in Teaching

	1995 RESPONSE % (*N*=339)	2005 RESPONSE % (*N*=464)
Strongly Agree	3.2	2.4
Agree	3.8	5.6
Neutral	4.7	9.7
Disagree	34.2	37.7
Strongly Disagree	53.1	37.9
No Response	1.0	6.7

The teachers were also asked whether they would like to use a calculator (1995) or graphic calculator (2005) more often, and 19% (1995) and 56.7% (2005) respectively said yes, a large increase over the ten years. Those who answered yes were asked to rank a number of obstacles, or add their own. Table 3.3 shows the results of these responses. It is clear that the major obstacle is still a lack of available calculators, but there has also been an increase in the need for professional development and greater teacher confidence. Given that 86% or more of students own their own calculator this is surprising. It may be that the lack of GC’s is what the teachers are talking about since only 27.1% own these.

Table 3.3 A Summary of Obstacles to Using the Calculator More in 1995 and 2005

OBSTACLES	% OF 1995 TEACHERS (*N*=64)		% OF 2005 TEACHERS (*N*=257)	
	FIRST MENTIONED	MENTIONED	FIRST MENTIONED	MENTIONED
Calculator Availability	76.6	81.3	52.5	71.6
Lack Of PD	4.6	12.5	19.1	48.2
Lack Of Confidence	4.7	10.9	13.6	42.4
Government Policy	1.6	9.4	1.9	6.2
School Policy	3.1	10.9	0	5.1

In 2005 these obstacles were also examined along gender lines to see if there were any differences (see Table 3.4). The results show that while females appeared a little less confident than males, this was not significant (χ^2=2.27, n.s.), and there were no other gender differences.

Table 3.4 A Summary by Gender of 2005 Obstacles to Using the Calculator More

Obstacles	First Mentioned (%)		Mentioned (%)	
	Male	Female	Male	Female
Calculator availability	30.2	31.0	42.2	40.5
Lack of PD	12.1	10.0	29.7	26.2
Lack of confidence	6.9	8.6	21.6	28.1
Government policy	1.7	0.5	9.1	3.8
School policy	0.0	0.0	7.3	2.9

The subject of sufficient resources was not raised here but was asked in a separate question, namely whether they agreed with the statement that a major obstacle to teachers using 'calculators or computers' (1995) or 'technology' (2005) is a 'lack of good ideas which work in the classroom' (1995) or 'classroom resources' (2005). While these questions are not precisely parallel they do show that the 41.0% agreeing in 1995 had increased significantly (χ^2=76.5, $p<0.0001$) to 71.1% in 2005, with a corresponding drop in those who disagreed from 32.2% to just 11.0%. Clearly the ten years have seen an even greater need for classroom resources with good ideas for teachers to use when teaching with technology. This is a surprising result given the large increase in calculator use around the world and the consequent increase in available resources, and may be, in spite of the internet, the result of poor communication of ideas. Whatever the reason it is something that educators need to be aware of and try to address.

Two of the open questions on the teacher questionnaire (Figure 2.1, Q's 22, 23) asked what the teachers perceived as the primary advantages and disadvantages of technology use, in order to try and get an idea of the motivation behind its use. There was a wide variety of responses to Q22 about the advantages of technology use and a summary of the number of occurrences of particular points mentioned is given in Table 3.5. Among these, improved efficiency of calculation (quicker calculations) was regularly mentioned, as was the benefit of visual explanation. Some teachers felt that students gained confidence through the use of technology, as they were able to check their solutions, spend less time on trivial manipulation, and eliminate careless errors, with calculators widely believed to provide 'efficient and accurate calculations and predictions'. Motivation was seen as another advantage with a response that technology, in the form of graphics calculators or computers 'can hook students' interest'. However, according to [4] these are the kinds of advantages seen by teachers who are new to technology use, and who have not made great progress in its implementation, or in personal instrumentation of the tools (see [6]). Those who have better PTK tend to perceive the mathematical benefits more. However, in this survey opinion was split about whether the use of technology aids understanding of mathematical concepts, and as Table 3.5 shows, understanding was only mentioned 37 times. One teacher had apparently moved to this point, saying that technology "allows [the] class to concentrate on [the] application of Maths techniques etc, rather than calculations, graph drawing, etc", while another responded that "traditional skills and techniques are being lost". It was also mentioned that technology use can prepare students for how the real world uses mathematics.

Table 3.5 Distribution of Types of Advantages Mentioned for Technology Use

Advantage	Frequency mentioned	% (*N*=257)
More efficient, quicker	149	32.0
Visualisation/Visual display	42	9.0
Student motivation/interest	39	8.4
Aids understanding	37	8.0
Improves confidence	14	3.0
Fewer errors in calculation	7	1.5

In the follow-up interviews with 32 teachers we asked why they used technology, and their answers were wide-ranging. It should be remembered that these were teachers who had used the technology and were confident enough to allow observation of their lessons by researchers. It was noticeable here that these teachers make more mention of ideas such as 'clarifies the concept', 'see the concept is really helpful', 'Technology is really important for multiple representations especially that graph/table/equation link', 'good way for making students understand the concepts better', and 'Allows students to investigate. Hands-on approach, particularly with problem simulation'. It appears that their increased confidence with the technology has enabled them to reach a level of PTK where they can think about the mathematics more.

In Q23 of the survey all the teachers were asked about the main disadvantages of technology use and Table 3.6 summarises these perceived disadvantages.

Table 3.6 Distribution of Types of Disadvantages Mentioned for Technology Use (*N*=257)

Disadvantage	Frequency	Mentioned (%)
Equipment – availability/quality/functionality/cost	93	20.0
Impedes learning/understanding	78	16.8
Dependence on calculator	58	12.5
Lack of confidence/knowledge–teachers or students	33	7.1
Time constraints	24	5.2
Distraction	11	2.4

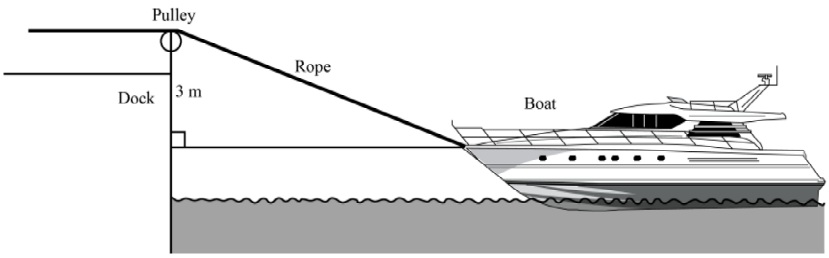
Interestingly, although, as we saw above, some said that the use of technology aided understanding, others said that it did the opposite. A common concern was that teachers thought that students are not gaining a full understanding of topics, and were instead relying on their calculators to tell them the answer. Also mentioned was how students are more likely to accept answers without considering how reasonable they are. One teacher said that graphical calculators "encourage kids to take short cuts, especially in algebra. Real algebra skills are lacking as a result" and 31 teachers mentioned that students often become very dependent on the calculator. This perceived effect of impedance of understanding was closely linked to a dependence on technology by many respondents. Some said they felt that the benefits of technology are small and often exaggerated, and that the technology should only used to support the primary content being taught. Some teachers also thought that technology is sometimes not appropriate, depending on what is being taught, and that teachers should not force the subject to fit the technology. Others believed that students may take advantage of lessons including technology, saying, for example, that it is "seen as an easy period by students".

Several teachers complained that an excessive amount of time is wasted when technology fails, and that sometimes not much learning takes place when students are distracted with some of the other things that technology can do. Varying standards of competence also cause difficulties in the classroom, with some students being highly skilled, while others are not. In summary, in spite of some comments above, based on the survey, we can infer that the teachers generally believe that there are benefits in using calculators in mathematics teaching.

3.1 Issues of assessment and equity

Since 1995 a new national assessment system has been introduced in New Zealand, called the National Certificate of Educational Achievement (NCEA). While a few schools opt for the International Cambridge examinations or the International Baccalaureate for their final year students, the vast majority of students sit NCEA internal assessments and external examinations at ages 16 (Level 1), 17 (Level 2) and 18 (Level 3). Each level of NCEA is divided into a number of standards, and clear guidance is given in the notes accompanying all of these that the use of appropriate technology is expected. In order to be able to place some of the comments below in context, Figure 3.1 shows examples of NCEA Level 3 questions on differentiation and integration. There are three levels of questions, and possible achievement: achieved; merit; and excellence. Heads of departments were asked (section A of the questionnaire) whether their department has a technology policy and what their departments do to implement technology in NCEA levels 2 and 3 mathematics teaching. Only 37.2% of the schools claimed to have a technology policy, and most of these were very basic, or general, such as "Technology should be used wherever possible as an aid to learning" or "To use technology as appropriate in lessons" or addressed procedural matters, along the lines of: "Room is locked. Supervision necessary (for students). Each student has own computer that they sit at (so any misuse can be quite easily narrowed down)"; and "Yes, students sign to agree to use gear appropriately and not access inappropriate sites on the web". Sometimes they involved technology types "Middle school committed to a laptop programme across all curriculum areas", "Our dept policy is that all students have access to (through purchase or hireage) and are proficient in the use of calculators" but only rarely did they address teaching or student learning, "Year 9 ICT access the curriculum policy – 4 weeks have planned for each class" and "ICT should be a tool used to assist students in their learning. All teachers are expected to integrate ICT into their teaching and learning practices". The majority of responses focused on the kinds of technologies used, with 46.9% identifying graphic calculator use in NCEA. It was clear from the survey responses that while teachers support the use of technology in many of the NCEA achievement standards, they do not believe that *all* of these should be supported with technology. One teacher expressed her concern this way:

> We are getting a mixed message from the NCEA examiners. The standard says 'appropriate technology' should be used but the Merit and Excellence questions are often designed to require algebraic manipulation, so we generally teach algebraic techniques for solving equations, knowing that weak girls will depend more on their calculators than strong ones.

Achieved	Differentiate the following functions. You do not need to simplify your answers. (a) $y=(x^2-3x)^5$ (b) $y=5\cot 2x$ (c) $y=\dfrac{\sin x}{x+3}$
Merit	A boat is pulled into dock by means of a rope running through a pulley on the dock. The rope is attached to the bow of the boat at a point 3 metres below the level of the pulley. The rope is being pulled through the pulley at a rate of 8 metres per minute.  At what rate will the boat be approaching the dock when there is 12 metres of rope between the boat and the pulley?
Excellence	A spherical balloon is being filled with air at a constant rate. At a certain time, the volume is 360 cm³. 20 seconds later, the volume is 450 cm³. Find the rate of increase of the surface area when the balloon has a volume of 1500 cm³.

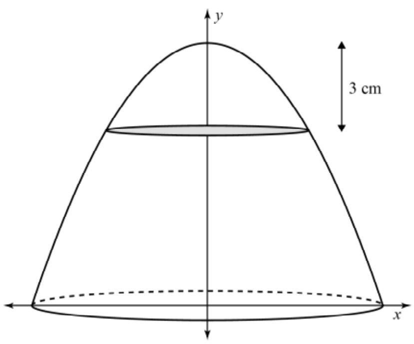
Achieved	Find the integrals. You do not need to simplify your answers. (a) $\int -\operatorname{cosec} 3x \cot 3x \, dx$ (b) $\int \frac{5}{3x} \, dx$
Merit	Assume that at any time the population of a town increases at a rate proportional to its population. Initially, the population of the town is 100 people. A year later, the population is 105 people. At this rate, how long does it take for the population to double?
Excellence	A paperweight on Oscar's desk contains coloured liquid. The inside of the paperweight, which contains the liquid, can be modelled by rotating part of the parabola $y=9-x^2$, from $x=0$ to $x=3$, through 360° about the y-axis.  When the paperweight is the correct way up, as shown above, the surface of the liquid is 3 cm below the top of the paperweight. Oscar tips the paperweight upside down. What will the depth of the liquid be now?

Figure 3.1 Sample differentiation and integration questions from NCEA Level 3 examinations (Some formatting changes - Questions obtained from New Zealand Qualifications Authority http://www.nzqa.govt.nz with permission.)

One reason for the ambivalence is that some teachers are not aware that in NCEA technology is expected in all standards. For example, many of the teachers (44.7%, mean agreement score 2.71 out of 5) disagreed when asked if technology use is expected in all NCEA standards, with only 26.3% agreeing or strongly agreeing. When asked whether "NCEA has too much emphasis on technology", only 11.7 % either agreed or strongly agreed, although those who disagreed or strongly disagreed did not reach a majority (44.2%), with 36.9% giving a 'neutral' response (mean agreement score 2.59 out of 5). When asked the reverse question of whether they believe that "NCEA has too little emphasis on technology", 43.3% either disagreed or strongly disagreed while a small number (6.2%) agreed (mean agreement score 2.52 out of 5). Hence the survey seems to indicate that the teachers believe that the NCEA assessment regime has the right level of emphasis on technology. This agreed with the list of achievement standards where technology was used, with the greatest use clearly in statistics and modelling.

When teachers were asked how NCEA had affected their teaching with technology, there were mostly positive responses about the change: "…it has been a positive thing… we've been able to write our own standards and activities using the basis of what we want to do…doing (NCEA) has increased our use of technology in terms of teaching… not that boring monotonous low skill stuff."; "NCEA has been really positive for technology in mathematics because…it says, students will use appropriate technology … if they're gonna do NCEA, they must use appropriate technology, and school's been really supportive and provided the money."; and "The beauty of NCEA is that you can now teach things properly. We use it [technology] a lot. It has increased our workload a lot, but it is far more valuable in terms of long term gain for students." However, there were some negative comments "I would love to see algebraic manipulations in schools. Students can get answers on graphics calculators but level of understanding has gone down as no working is required. NCEA is destroying maths." The depth of feeling some have on this topic can be seen in the comment of one teacher who said that "NCEA encourages [us] to teach students to get answers only (working is not marked) to questions they do not understand by learning which buttons to press, on a piece of technology that nobody outside a classroom uses, and which will be out of date within 3 years."

A recent proposed innovation in New Zealand is the intended introduction of computer algebra system calculators (CAS) in schools. There has been a lot of international research in recent years on the perceived benefits of using these calculators in the mathematics classroom (see e.g., [3], [11], [16], [17] and [18]). From our survey we knew that only 1.8% of the teachers used CAS with their classes, but we wanted to know whether teachers are in favour of their use in examinations, since this raises the problem of how to set questions that are both equitable and still test the required knowledge. Some research on this has been conducted (see [14]) and showed that, while it is possible to set examination questions that are equitable, there are considerations in terms of the possible disadvantages to weaker students of using CAS calculators. In addition [19], [20] found that students use some CAS syntax in examination solutions and have proposed the rubric RIPA (*Reasons–Inputs–Plan–(some)Answers*) as a guide for teaching students how to record their calculator solutions in examinations. The responses to the question whether 'All types of calculators should be allowed in examinations' showed that while 21.7% are in favour of this move, there is a sizeable majority of 60.5% who disagree. Currently GC's are allowed in the examinations. An occasional teacher comment expressing concern supports the view that there is some opposition to CAS implementation.

> The push for CAS concerns me as we will basically be rewarding the use of technology not assessing students on their ability in applying or using maths skills. What would NCEA at [Achieved] level really be worth if answers are generated on CAS?
>
> At this stage I would be very much against this change [CAS introduction] but I have an open mind and would be keen to hear the arguments both ways once they have been tested.
>
> I worry, that with increasing use of CAS calculators...The logical processes will be lost and hence only hi-flyers will see its benefit and be able to succeed.

Since the New Zealand Ministry of Education is moving towards allowing CAS in examinations from 2011/12 it seems that there is work to do to provide the professional development that will convince many teachers of the value of this. No doubt the outcomes of this move to CAS will be observed closely by educators in other countries.

In the 32 teacher interviews, most said that they use GC's for internal assessment, and that their students also use GC's for external examinations. One of the questions in the interviews with the 32 teachers using technology asked: 'Are there any equity or cultural issues you can see with the use of technology?' It was clear from the responses that there was just one issue that the teachers could see, namely the inequity arising from the fact that some students could not afford their own calculators, and schools were often not able to purchase them either, and this was seen as crucial during examinations. Typical responses were: "There are certainly equity issues among students that come from poorer homes where they can't afford them. I think that is probably going to be very, very difficult for schools in the lower decile areas."; "A lot of our students will come from even low decile areas… when you are asking for another $75,…for a graphic calculator, it's just nah, it's not gonna happen."; "…to be fair, I think the exam has to be designed in a way in which they can still test the manual understanding so that the students can only really rely on the graphics calculator to a certain extent"; and "I worry about the results indicating that that kid knows more in that assessment than a kid without a graphic calculator. When in fact the other kid may know more about maths and have a better understanding but they've run out of time and they've never had a chance to show what they know". To remedy this problem, two of the teachers said that they loan the school's calculators to the students during examination periods. One teacher said that, in order for the students to be able to use calculators in externals, the school buys in bulk and sells them to the students at a price cheaper than the retail price. However, another teacher replied that they stopped loaning their calculators for the reason that some of the calculators were not returned to the school. There were one or two teachers who expressed the opinion that there is no equity problem, noting that the GC's are cheaper than cellphones and stating "I went to the mid-year exams when the seniors are all doing their exam hall full of people there were so many graphics calculators I don't think there is even an equity issue. Maybe one or two kids…". Suggestions were made for ways to ameliorate the situation too, including encouragement to parents to see the GC as a family investment, to be passed on to younger siblings, pushing for larger department technology budgets, and recognising that in the end the responsibility rests with governments, "Then the Ministry of Education should be supplying funding for families so that they can have the same technology available for the education of their children". In summary, the only equity issue of concern raised by the teachers in the study was that of affordability of, and hence equality of access to, calculators. However, there was little evidence that this was perceived as a major problem.

4. Conclusion

In summary this study has shown that the number of calculators in schools increased during the period 1995 to 2005, but only 27% are GC's. Teachers claim that more are needed, even though around 86% of students have a calculator and regular use by teachers has fallen. The evidence is that teachers are generally still in favour of the use of calculators in the learning of mathematics, and see that there are benefits to doing so. However, a significant minority of teachers (27%) think that using calculators can be detrimental to student understanding of mathematics, depending on how they are used. Their fear is that students will become dependent on them and lose their by-hand skills. In spite of this a majority (56.7%) of teachers would like to use calculators more often in their teaching, and prefer the students to have their own, although there are equity issues over cost associated with this for schools in poorer areas. The major obstacles to such increased use are availability of calculators, relevant professional development, suitable classroom resources and teacher confidence. The lack of resources has increased over ten years and this study has shown a continuing need for high-quality classroom-based resources and the corresponding professional development to make good use of them. This professional development should specifically address the integration of technology into mathematics teaching in a manner that develops teachers' PTK by focussing on the mathematics more than the technology. Another concern with regard to increased use of calculators is that HOD's were concerned about teachers' knowledge of the technology, their confidence in using it for teaching, and the possibility of teacher resistance to its use in teaching.

Generally the teachers in the study were happy with the level of emphasis on calculators in the national assessment system (NCEA) and were positive about it. However, it should be noted that while the NCEA says that technology use is *expected*, this is not enforced in any way and neither are questions requiring calculators set in examinations. When it comes to moving beyond the status quo, where GC's are allowed in examinations, 60.5% of teachers were opposed to the use of all calculators in examinations, presumably referring to those with a CAS facility. When it comes to the proposed use of CAS in external examinations some teachers are excited by the possibilities for mathematics learning they believe this will open up, and others have exactly the opposite view, that it will destroy mathematics, especially algebraic manipulation and understanding of ideas. The issue of equity of access to calculators (probably GC's) was of special concern with regard to examinations due to the relatively high cost and hence affordability.

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